The abundance of relativistic axions in a flaton model of Peccei-Quinn symmetry

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Abstract

Flaton models of Peccei-Quinn symmetry have good particle physics motivation, and are likely to cause thermal inflation leading to a well-defined cosmology. They can solve the μ problem, and generate viable neutrino masses. Canonical flaton models predict an axion decay constant $F_{\rm PQ} \sim 10^{10}$ GeV and generic flaton models give $F_{\rm PQ} \gtrsim 10^9$ GeV as required by observation. The axion is a good candidate for cold dark matter in all cases, because its density is diluted by flaton decay if $F_{\rm PQ} \gtrsim 10^{12}$ GeV. In addition to the dark matter axions, a population of relativistic axions is produced by flaton decay, which at nucleosynthesis is equivalent to some number δN_ν of extra neutrino species. Focussing on the canonical model, containing three flaton particles and two flatinos, we evaluate all of the flaton-flatino-axion interactions and the corresponding axionic decay rates. They are compared with the dominant hadronic decay rates, for both DFSZ and KSVZ models. These formulas provide the basis for a precise calculation of the equivalent δN_ν in terms of the parameters (masses and couplings). The KSVZ case is probably already ruled out by the existing bound $\delta N_\nu \lesssim 1$. The DFSZ case is allowed in a significant region of parameter space, and will provide a possible explanation for any future detection of nonzero δN_ν.

1 Introduction

With the discovery of the instantons it was realized that the $\theta_{\rm QCD}$ parameter of the Standard Model can have important physical consequences. In particular, the induced CP violation affects the electric dipole moment of the neutron, leading to the upper limit limit $\theta_{\rm QCD} \le 10^{-10}$. An attractive explanation for such a small parameter is provided by the Peccei-Quinn mechanism [1, 2]. There is supposed to be a spontaneously broken global $U(1)$ symmetry (PQ symmetry), which is also explicitly broken by the color anomaly. Its pseudo-Goldstone boson is the axion. The PQ symmetry is spontaneously broken by some set of scalar fields ϕ_i (elementary or composite) with charges Q_i, so that their PQ transformation is

$$\phi_i \to e^{iQ_i\alpha}\phi_i \,. \tag{1}$$

Denoting the vacuum expectation value $\langle|\phi_i|\rangle$ by $v_i/\sqrt{2}$, we define the PQ symmetry breaking scale $F_{\rm PQ}$ by

$$F_{\rm PQ}^2 = \sum_i Q_i^2 v_i^2 \,. \tag{2}$$

(In defining $F_{\rm PQ}$, we use the canonical normalization of the PQ charges, that the smallest Q_i^2 is set equal to 1.) Collider and astrophysics constraints require roughly [3]

$$F_{\rm PQ}/N \gtrsim 10^9\,{\rm GeV} \,. \tag{3}$$

The bound is actually one on the axion mass, whose relation to $F_{\rm PQ}$ is given by

$$m_{\rm a} = 6N \times 10^{-4}\frac{10^{10}\,{\rm GeV}}{F_{\rm PQ}}\,{\rm eV} \,. \tag{4}$$

Here, N is the number of distinct vacua, or equivalently the number of domain walls meeting at each PQ string.

PQ charge will also be carried by fields which do not spontaneously break PQ symmetry. We shall consider KSVZ (hadronic) models [4] in which these are only some extra heavy quark superfields, and DFSZ [5] models in which they are only Standard Model (SM) superfields.

We are concerned[1] with models in which the fields breaking PQ symmetry are flatons. Flatons are fields whose tree-level potential is flat in the limit of unbroken renormalizable supersymmetry, which acquire nonzero vevs after soft supersymmetry breaking. We make the usual assumption, that the supersymmetry breaking is gravity-mediated.

In the rest of this section we recall the essential features of flaton and non-flaton models of PQ symmetry. In the next section we discuss in some detail the cosmology of flaton models. In Section 3, we give the general structure of the flaton and flatino masses. In Section 4 we analyze the general self interactions between flatons and flatinos. In Section 5 we see the effect of the interaction of the flatons with the matter fields. In Section 6 we consider the implication of the nucleosynthesis bound on the energy density of relativistic axions, in particular, of a possible future bound $\delta N_\nu < 0.1$. We conclude in Section 7.

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1.1 Models of PQ symmetry-breaking

Let us consider the potential of the fields ϕ_i which break PQ symmetry. In a supersymmetric model there have to be at least two, but for first orientation we pretend that there is only one. In the limit of exact PQ symmetry, its potential will be of the form

$$V = V_0 - m^2|\phi|^2 + \frac{1}{4}\lambda|\phi|^4 + \sum_{n=1}^{\infty} \lambda_n \frac{|\phi|^{2n+4}}{M_{\rm Pl}^{2n}} \,. \tag{5}$$

$M_{\rm Pl} = (8\pi G_N)^{-1/2} = 2.4 \times 10^{18}\,{\rm GeV}$ is the reduced Planck mass.

1.1.1 Non-flaton models

If the renormalizable coupling λ is of order 1, the non-renormalizable terms are negligible and the mass of the radial oscillation is $m \sim F_{\rm PQ}$. (Recall that $F_{\rm PQ} = \sqrt{2} Q\langle|\phi|\rangle$.)

Going on to the case where a number of fields ϕ_i ($i = 1, \cdots, p$) break PQ symmetry, there will be p particles corresponding to the radial oscillations, and (in addition to the axion) $p-1$ particles corresponding to the angular oscillations. There will also be p superpartners with spin 1/2. In a non-supersymmetric theory all of these particles would have mass of order $F_{\rm PQ}$, but supersymmetry protects the mass of one scalar and one spin-1/2 particle.

Indeed, in the limit of unbroken supersymmetry, the holomorphy of the superpotential ensures that PQ symmetry is accompanied by a symmetry acting on the radial parts of the PQ charged fields

$$\phi_i \to e^{Q_i\alpha}\phi_i \,. \tag{6}$$

The corresponding pseudo-Goldstone boson is called the saxion (or saxino), and the spin-1/2 partner of the axion/saxion is called the axino. With gravity-mediated supersymmetry breaking, the saxion and axino will typically both have soft masses of order 100 GeV, although specific models exist [6] with an axino mass of order keV. In gauge-mediated models, the saxion and axino will typically both have the sub-keV mass.

In non-flaton models one can hope to understand a value $F_{\rm PQ} \sim 10^{10}\,{\rm GeV}$ since that is the supersymmetry breaking scale [7], but it may be hard to understand a bigger value.

1.1.2 Flaton models

We are concerned with models [8, 9, 10, 11, 12] in which the fields breaking PQ symmetry are flaton fields [13]. This means that their tree-level potential is flat in the limit of unbroken, renormalizable supersymmetry. We assume that supersymmetry breaking is gravity-mediated, which is usual in the context of flaton fields.

Considering first the case of one flaton with potential Eq. (5), the quartic term is (practically) zero, while the mass m of the flaton is of order 100 GeV to 1 TeV. When making numerical estimates, we shall take for definiteness

$$m = 10^{2.5\pm0.5}\,{\rm GeV} \,, \tag{7}$$

The leading non-renormalizable term generates a large vacuum expectation value (vev),

$$F_{\rm PQ} = \sqrt{2}\langle|\phi|\rangle \sim \left(m M_{\rm Pl}^n/\sqrt{\lambda_n}\right)^{\frac{1}{n+1}} . \tag{8}$$

Here, λ_n is the coefficient of the leading term in Eq. (5), which is expected to be roughly of order 1. In the case $n=1$ this gives a vev of order $10^{10}\,$GeV, but it can be bigger if n is bigger. Imposing the condition that V (practically) vanishes in the vacuum gives the height of the potential,

$$\left(\frac{V_0^{1/4}}{10^6\,{\rm GeV}}\right) \sim \left(\frac{\langle|\phi|\rangle}{10^{10}\,{\rm GeV}}\right)^{1/2} . \tag{9}$$

We are concerned with the case that the ϕ_i are flaton fields. The scalar particles are now the axion, plus $2p-1$ flatons with mass of order $100\,$GeV, and p flatinos with masses of the same order. The saxion, defined through Eq. (6), is a linear combination of flatons, while the axino (defined as the partner of the axion-plus-saxion) is a linear combination of flatino mass eigenstates. Neither of them has any special significance. In particular, the possibility of a keV-mass axino does not exist in flaton models. In the models that we shall consider, $p=2$ so that there are three flatons and two flatinos.

1.2 Estimates of $F_{\rm PQ}$

To estimate the magnitude of $F_{\rm PQ}$ in flaton models, we begin with schematic case of one flaton field, with potential Eq. (5).

Consider first the expected magnitude of the coefficients λ_n in Eq. (5). In the case that $M_{\rm Pl}$ represents the ultra-violet cutoff for the effective field theory, one usually assumes $\lambda_n \sim 1$, but $\lambda_n \sim 1/(2n+4)!$ may be more realistic [14]. On the other hand, one might quite reasonably suppose that the cutoff, call it $\Lambda_{\rm UV}$, is around the gauge coupling unification scale $10^{-2}M_{\rm Pl}$ (either because fields of a Grand Unified Theory have been integrated out, or because this is the true quantum gravity scale). In that case, the estimate $\lambda_n \sim 1/(2n+4)!$ to 1 would be reasonable if $M_{\rm Pl}$ were changed to $\Lambda_{\rm UV}$ in Eq. (5). Retaining $M_{\rm Pl}$, one should multiply the estimate of λ_n by a factor $(M_{\rm Pl}/\Lambda_{\rm UV})^{2n} \sim 10^{4n}$. In view of these considerations, we adopt as a reference the estimate $1/(2n+4)! \lesssim \lambda_n \lesssim 10^{4n}$, corresponding to $\lambda_1^{\frac{1}{4}} = 10^{0.2\pm0.8}$ and $\lambda_2^{\frac{1}{6}} = 10^{0.3\pm1.0}$.

Using these estimates of λ_n, we can make estimates of $F_{\rm PQ}$ bearing in mind the uncertainty Eq. (7) in m. In these and other estimates, we add in quadrature different uncertainties in the exponents. This procedure has no particular basis, but at least it is better than ignoring the uncertainties completely, or adding different estimates linearly. In the case at hand, the uncertainty is dominated by the large uncertainty that we assigned to the λ_n. We take the PQ charge of ϕ to be 1, so that $F_{\rm PQ} = \sqrt{2}\langle|\phi|\rangle$.

Unless it is forbidden by a symmetry, the leading term $n=1$ will be the one appearing in Eq. (8), leading to

$$F_{\rm PQ} = 10^{10.4\pm0.9}\,{\rm GeV} \qquad (n=1) . \tag{10}$$

If the leading term is $n = 2$ this becomes $F_{\rm PQ} = 10^{12.9\pm1.1}\,\rm GeV$. At higher n, $F_{\rm PQ}$ slowly increases, so for $n > 1$ we have

$$F_{\rm PQ} > 10^{11.8}\,\rm GeV \qquad\qquad (n > 1)\,. \tag{11}$$

In the class of supersymmetric models that we shall consider, PQ symmetry is broken by two flaton fields $\phi_{\rm P}$ and $\phi_{\rm Q}$, with charges respectively 1 and $2n+1$. The fields interact, giving the rather complicated potential Eq. (30) for $n = 1$ and one of similar form [15] for bigger n. In all cases, the vevs of both flaton fields are given roughly by Eq. (8). There are additional uncertainties because there are more soft parameters and factors involving n, but in view of the large uncertainty we already assigned to the coupling λ_n Eqs. (10) and (11) should still provide reasonable estimates.

2 Cosmology and dark matter

2.1 Cosmology of the PQ fields

Generic models of PQ symmetry have many possible cosmological consequences, which have been discovered gradually over the years.

In all cases the axion lifetime is longer than the age of the Universe [2], so that its present density must be $\Omega_{\rm a} \leq 0.3$ or so, with the equality prevailing if axions are the dark matter. The density depends on whether the axions come from strings or from the vacuum fluctuation of the axion field during inflation. (In the latter case the axion density [16] depends on our location in the universe.) Strings may be produced by a variety of mechanisms during [17] and after inflation. The axion density depends also on the amount of any entropy production after the axion mass switches on at a temperature around 1 GeV. As a result there is no model-independent prediction for $\Omega_{\rm a}$ as a function of $F_{\rm PQ}$.

Within the usual framework of non-flaton models, the superpartners of the axion can also have a range of cosmological consequences. An axino with a keV mass is a dark matter candidate, which may be produced by a variety of mechanisms and give rise to a variety of cosmological consequences [18, 6]. Alternatively, an axino with a 10 GeV mass may be the cold dark matter [19], as it can be the lightest supersymmetric particle (LSP). Finally, the saxion is a late-decaying particle which may be produced by thermal or other mechanisms. If it is sufficiently abundant to dominate the density of the Universe, it must decay well before nucleosynthesis, and before it does so it will dilute the abundance pre-existing relics, including baryons and dark matter candidates [20, 21]. If it is less abundant it may decay much later, and affect the formation of large-scale structure [22].

In contrast with this generic situation, the cosmology of flaton models is rather well-defined, on the reasonable assumption [23, 15] that the PQ flaton fields generate an era of thermal inflation [24, 25, 23, 26, 27, 28], which is not followed by any other such era. Thermal inflation occurs long after the ordinary inflation which is supposed to be origin of structure, and may wipe out all previously existing relics. When it ends, PQ strings are produced, and flaton decay (the analogue of saxion decay) produces a calculable amount of

entropy, with the reheating at a calculable temperature in the range MeV to TeV. As a result, the axion density is in principle calculable, and appears to be compatible with the observed dark matter density for any $F_{\rm PQ}$ allowed by other considerations (Section 2.3.) In other words, the axion is a good dark matter candidate in flaton models. Finally, the flatinos (the generalizations of the axino) cannot have the keV mass, and for simplicity we assume that none of them is the LSP. (The opposite case will be explored in a future paper [29].)

A unique feature of flaton models is that flaton decay creates a highly relativistic population of axions [23, 15]. This population has nothing to do with the dark matter. Its density at nucleosynthesis is equivalent to roughly $\delta N_\nu \sim 1$ extra neutrino species [15]. Flaton models of PQ symmetry will therefore be a candidate for explaining a nonzero δN_ν that may be established in the future, and the models will be strongly constrained if the present bound [30] $\delta N_\nu \lesssim 1$ is significantly tightened.

2.2 Thermal inflation and reheating

In the early Universe, with Hubble parameter $H \gtrsim 100\,\mathrm{GeV}$, fields with the true soft mass $|m| \sim 100\,\mathrm{GeV}$ are expected [31] to have an effective mass-squared $m^2(t) \sim \pm H^2$. (During inflation this result might be avoided [32], but it should still hold afterwards [33, 34, 35].) This applies in particular to the flaton fields. Pretending for the moment that there is only one flaton field, we focus on the case that $m^2(t)$ is positive in the early Universe, because thermal inflation [24, 25, 23, 26, 27, 28] then occurs, leading to rather definite predictions for the cosmology.

Let us summarize the history. After H falls below $|m|$, there will be enough thermalization to hold ϕ at the origin until thermal inflation begins. (See Section IIIB of [23].) Thermal inflation begins when the potential V_0 dominates the energy density. This is at the epoch $T \sim V_0^{1/4} \sim 10^6\,\mathrm{GeV}$ (assuming for simplicity that full reheating has occurred by that time). Thermal inflation ends after $\sim \ln\left(V_0^{1/4}/|m|\right) \lesssim 10$ e-folds, when the temperature is of order $|m| \sim 100\,\mathrm{GeV}$.

When thermal inflation ends, the flaton field ϕ moves away from the origin. Cosmic strings form, and between them the roughly homogeneous flaton field starts to oscillate around its vev. Corresponding to the oscillation is a population of flatons. We discuss in Section 2.4 the possibility that parametric resonance rapidly drains away the energy of this oscillation, finding that this phenomenon will probably not occur and will in any case have little effect on the following considerations. Discounting parametric resonance, energy loss comes at first only from the Hubble drag, which is negligible during one oscillation. The oscillation corresponds to flatons, with conserved number and non-relativistic random motion (matter as opposed to radiation). When the flatons decay, the Universe thermalizes, at the reheat temperature [23]

$$
\begin{aligned}
T_{\rm RH} &\simeq 10^{0.3} g_{\rm RH}^{-\frac{1}{4}} \sqrt{M_{\rm Pl}\Gamma} \simeq 10^{-0.2}\sqrt{10^{-2} M_{\rm Pl} c N_{\rm chan} m^3 F_{\rm PQ}^{-2}} && (12) \\
&= 10^{2.2\pm0.5} \left(\frac{10^{10}\,\mathrm{GeV}}{F_{\rm PQ}}\right)\left(\frac{m}{10^{2.5}\,\mathrm{GeV}}\right)^{3/2} \mathrm{GeV}\,. && (13)
\end{aligned}
$$

In the first expression, Γ is a typical flaton decay rate, while $g_{\rm RH} \sim 100$ is the effective number of particle species at $T_{\rm RH}$. In the second expression $N_{\rm chan}$ is the number of decay channels, c is a factor of order 1 and m is a typical soft parameter. The factor 10^{-2} in the estimate of Γ is what one expects in the case of unsupressed couplings for a single decay channel [23], and it is confirmed by, for instance, the estimates in Eq. (54), *etc.* There are in reality several channels, and for definiteness, we take $cN_{\rm chan} = 10^{1.0\pm1.0}$, leading to the estimate in the third line.

Adding in quadrature the uncertainty of Eq. (7), we find

$$T_{\rm RH} = 10^{2.2\pm0.9} \left(\frac{10^{10}\,{\rm GeV}}{F_{\rm PQ}} \right) {\rm GeV}\,. \tag{14}$$

In order not to upset nucleosynthesis, it is required to have $T_{\rm RH} \gtrsim 10\,{\rm MeV}$, which corresponds to

$$F_{\rm PQ} \lesssim 10^{15}\,{\rm GeV}\,. \tag{15}$$

Using Eqs. (10), (11), (7) and (13), we estimate

$$T_{RH} = 10^{1.8\pm1.3}\,{\rm GeV} \qquad (n=1) \tag{16}$$

$$T_{RH} \lesssim 10^{0.4}\,{\rm GeV} \qquad (n>1)\,. \tag{17}$$

In this discussion of thermal inflation and its aftermath, we have retained the pretense that there is just one flaton field. There are in the models we shall consider two flaton fields ϕ_P and ϕ_Q, with the potential Eq. (30) or its $n>1$ analogue. We assume that $m_{\rm P}^2(t)$ is positive in the early Universe, so that thermal inflation occurs. When thermal inflation ends, $\phi_{\rm P}$ moves away from the origin, and as a result $\phi_{\rm Q}$ also moves away from the origin. At first the orbit in field space will be far from the vev, but after a few Hubble times the Hubble drag will allow the vev to attract the orbit towards it, so that there are almost sinusoidal oscillations of the eigenmodes around their vacuum values. These oscillations are equivalent to the presence of the three species of flaton, and each of them decays at the epoch specified by Eq. (12), with m the appropriate mass. The reheating process is complete after the last decay has taken place.

2.3 Dark Matter and baryons

Axionic dark matter

The axion number density is conserved after some epoch $T_{\rm cons} \sim 1\,{\rm GeV}$. In the case $n=1$, $T_{\rm RH}$ is bigger than $T_{\rm cons}$ and entropy is conserved too. The axion density is then expected to be of the form

$$\Omega_{\rm a} = C \left(\frac{F_{\rm PQ}}{10^{12}\,{\rm GeV}} \right)^{1.2}\,. \tag{18}$$

The constant C is in principle calculable from the dynamics of the strings, walls and axions, derived ultimately from the field equation of the flaton fields breaking PQ symmetry. According to one group [36, 37, 38], $C \sim 1$ to 10, while according to another [39, 40, 41, 42],

$C \sim 0.2$. In view of this uncertainty, which we emphasize is one of computation rather than principle, we conclude that the axion is a good dark matter candidate in the case $n = 1$ which corresponds to Eq. (10). (By a 'good' candidate, we mean one whose density is predicted to be within at least a few orders of magnitude of the observed dark matter density.)

In the case $n > 1$, $T_{\rm RH}$ is smaller than $T_{\rm cons}$. Entropy is produced until the epoch $T_{\rm RH}$, giving [15, 43]

$$\Omega_{\rm a} = \tilde{C} \left(\frac{10^{12}\,{\rm GeV}}{F_{\rm PQ}} \right)^{0.44} , \tag{19}$$

with roughly $\tilde{C} \sim 10C$. At least if C is not too big, the axion is a good dark matter candidate in these models too [15].

Baryogenesis and the LSP

If thermal inflation wipes out pre-existing relics, baryogenesis has to occur after thermal inflation. The crucial factor here is the final reheat temperature $T_{\rm RH}$. Baryogenesis mechanisms occurring at the electroweak phase transition can operate if $T_{\rm RH} \gtrsim 100\,{\rm GeV}$. This is possible in the canonical case $n = 1$, but not in the case $n > 1$. If $T_{\rm RH}$ is smaller one must turn to other mechanisms, which are quite speculative. Proposals include a complicated Affleck-Dine mechanism along the lines of [27], QCD baryogenesis [44] or parametric resonance baryogenesis [45].

If the lightest supersymmetric particle (LSP) is stable, it has to thermalize in order to avoid overproduction from flaton decay. This requires $T_{\rm RH} \gtrsim m_{\rm LSP}/20$ [15], say $T_{\rm RH}$ more than a few GeV. This is exactly what one expects in the case $n = 1$. As is well known, a stable LSP is a good dark matter candidate.

If the LSP is unstable (due to R-parity violation), baryogenesis can occur simply by allowing a baryon-number violating flaton decay channel (the Dimopoulos-Hall mechanism [46]). This mechanism requires DSFZ as opposed to KSVZ coupling to matter, and as we shall see the former case is favored in flaton models. For the mechanism to work, final reheat must occur at a temperature less than a few GeV [46]. This is likely for $n > 1$, but looks rather unlikely in the canonical case $n = 1$.

Let us summarize. In the canonical case $n = 1$, the LSP can thermalize, and therefore can be stable so that it is a good dark matter candidate just like the axion. In this case, baryogenesis mechanisms involving the electroweak phase transition can operate. In the case $n > 1$, the LSP cannot thermalize and therefore cannot be stable. Baryogenesis from flaton decay (the Dimopoulos-Hall mechanism) is a natural possibility in this case.

Supermassive dark matter

We have seen that the axion is a good dark matter candidate, and that in the canonical model the LSP is also a good candidate. These conclusions hold both in the DFSZ and KSVZ cases. In the KSVZ case, there is a third good dark matter, namely the heavy quarks E, E^c, which are strongly interacting massive particles (SIMPs). Until thermal inflation ends, the SIMPs are light and will be in thermal equilibrium. After thermal inflation ends,

SIMPs acquire mass through the coupling $\phi E E^c$ to the flaton. The density of such particles has been shown [28] to be naturally in the right ballpark.

The scenario of [28] should be contrasted with an earlier proposal [47], that the supermassive particle is very heavy also in the early Universe, and never in thermal equilibrium. Such a particle may be produced by the vacuum fluctuation during ordinary inflation [47], or by other mechanisms. In the former case, each comoving wavenumber leaving the horizon during inflation will acquire very roughly one particle per quantum state [48, 49, 50], leading very roughly to [49, 47, 50]

$$\Omega_0 = \left(\frac{m}{10^{14}\,\mathrm{GeV}}\right)\left(\frac{H_*}{10^{14}\,\mathrm{GeV}}\right)\left(\frac{T_{\mathrm{INFRH}}}{10^{9}\,\mathrm{GeV}}\right)\gamma\,. \tag{20}$$

In this expression, m is the mass of the superheavy particle, H_* is the Hubble parameter during slow-roll inflation, and T_{INFRH} is the temperature at reheat after slow-roll inflation, and γ is the dilution caused by entropy production after that epoch. In our case, thermal inflation will give roughly $\gamma \sim e^{-10}$. One can adjust the other parameters to make $\Omega_0 \sim 1$, but in contrast with the LSP and the axion the required value $\Omega_0 \sim 1$ is not favored over any other. In other words, this kind of supermassive dark candidate is not (in our present state of knowledge) a *good* dark matter candidate, merely a possible one.[2]

2.4 Parametric resonance?

To check whether parametric resonance [51] occurs, we make the very crude approximation that the sinusoidal oscillation corresponding to the three flatons is present from the very beginning. We also assume that the masses of the three flatons are roughly the same, or else that one of the amplitudes is much bigger than the others. Then the field equation of the Fourier component of each produced field ϕ_n is a Mathieu equation, leading to a situation that has been analyzed in the literature [51]. (More realistic cases, including the one where the oscillation starts at a maximum of the potential [52], seem to give similar results.) The oscillation of the real field ϕ_I corresponding to the Ith flaton leads to an oscillating mass-squared $m_n^2(\phi_I)$ for each of the produced scalar fields. Parametric resonance occurs, leading to significant production of ϕ_n, if [53]

$$q \equiv m_{0n}^2(\phi_I)/m^2 \gtrsim 10^3\,. \tag{21}$$

In this formula, $m \sim 100\,\mathrm{GeV}$ is a typical oscillation frequency, and $m_{0n}^2(\phi_I)$ is the amplitude of $m_n^2(\phi_I)$. The initial amplitude of oscillation is $\phi_{I0} \sim F_{\mathrm{PQ}}$. The scalar particles that can be produced include the flatons and the axion, and in the DFSZ case also the Standard Model Higgs and sfermions. From Sections 4 and 5, each of these has $m_{0n} \sim 100\,\mathrm{GeV}$, making $q \sim 1$. The conclusion is that parametric resonance probably does not occur, but a more

[2] The same is true in the case $\gamma = 1$. In particular, the choice $m \sim H_*$ advocated in [47] is not in fact particular favored, bearing in mind that m is the true mass as opposed to the effective mass during inflation. Inflation with a potential $V = \frac{1}{2}m_{\mathrm{INF}}^2\phi_{\mathrm{INF}}^2$ indeed ends when $H \sim m_{\mathrm{INF}}$, but the inflaton field ϕ_{INF} is not supposed to be stable and hence is not a dark matter candidate.

detailed calculation is needed to say anything definite especially in view of the extremely anharmonic nature of the initial oscillation.

Even if it occurs, the overall effect of parametric resonance will not be dramatic, unless it leads to baryogenesis [45]. Its initial effect is to quickly damp the flaton field oscillation, producing flatons, axions and in the DFSZ case Higgs and sfermions. The produced particles have very roughly the same energy density, and are marginally relativistic except for the axions which are highly relativistic. The flatons are stable on the Hubble timescale while the Higgs and sfermions decay into highly relativistic ordinary matter (plus the marginally relativistic LSP if it is stable). After a small number of Hubble times the energy in the highly relativistic particles becomes negligible. The dominant energy is in non-relativistic flatons, coming partly from the parametric resonance and partly from the residual homogeneous oscillation of the flaton fields. Except for the baryogenesis possibility, there is no change.

2.5 Relativistic axions

Now we come to the relativistic axions, which will be our concern for the rest of the paper. This axion population comes from the decay of the flatons [23, 6] when they finally reheat the Universe. Its density during nucleosynthesis is conveniently specified by the equivalent number of extra neutrino species

$$\delta N_\nu \equiv \left(\frac{\rho_{\rm a}}{\rho_\nu}\right)_{NS}, \tag{22}$$

where $\rho_{\rm a}$ is the energy density of relativistic axions, and ρ_ν is the energy density of a single species of relativistic neutrino.

At present, constraints coming from nucleosynthesis are bedeviled by the fact that there are two separate allowed regions of parameter space, corresponding to 'low' and 'high' deuterium densities. In the 'high' region, the bound [30] at something like 2-σ level is $\Delta N_\nu < 1.8$, while in the perhaps favored 'low' region, the bound at a similar level is $\Delta N_\nu < 0.3$. As we shall see, the flaton models predict δN_ν roughly of order 1, so that at least the second bound is quite constraining.

In the canonical model that we shall discuss, there are three flaton species, and in more general models there are more flatons. In general, each flaton species can decay into relativistic hadronic matter X, into Xa where a denotes a relativistic axion, or into aa. (We neglect for simplicity the small branching ratio into channels containing more axions.) Let us pretend first that there is one flaton ϕ. The hadrons X thermalize immediately, but the axions do not thermalize [15], so their density at reheating is

$$\rho_{\rm a} = B_{\rm a}\rho_{\rm r}\,, \tag{23}$$

where

$$B_{\rm a} \equiv \frac{\Gamma(\phi\to {\rm aa}) + \frac{1}{2}\Gamma({\rm I}\to {\rm Xa})}{\Gamma(\phi\to {\rm X})}\,, \tag{24}$$

and the density of thermalized radiation is

$$\rho_{\rm r} = \frac{\pi^2}{30} g_{\rm RH} T_{\rm RH}^4 \,.$$

After reheating, $\rho_{\rm a} \propto (a_{\rm RH}/a)^4 = (s_{\rm NS}/s_{\rm RH})^{\frac{4}{3}}$, where a is the scale factor, and s is the entropy density of the particles in thermal equilibrium. The latter is given by $s = (2\pi^2/45) g T^3$, where g is the effective number of relativistic species in thermal equilibrium and T is their temperature. At the beginning of the nucleosynthesis era, $g = 10.75$ and

$$\rho_{\rm a} = B_{\rm a} \frac{\pi^2}{30} g_{\rm RH} \left(\frac{10.75}{g_{\rm RH}} \right)^{\frac{4}{3}} T_{\rm NS}^4 \,. \tag{25}$$

The density of one neutrino species at that epoch is

$$\rho_\nu = \frac{\pi^2}{30} \frac{8}{7} T_{\rm NS}^4 \,,$$

so that

$$\delta N_\nu \equiv 13.6 g_{\rm RH}^{-\frac{1}{3}} B_{\rm a} = 2.9 B_{\rm a} \,. \tag{26}$$

For the last equality we used $g_{\rm RH} = 100$, appropriate if $T_{\rm RH} \sim 1\,{\rm TeV}$. The alternative choice $g_{\rm RH} \simeq 10$ would reduce δN_ν by a factor 2 or so.

In models with more flatons, denoted by a label I, the quantity $B_{\rm a}$ to be used in Eq. (26) is given by

$$B_{\rm a} \equiv \frac{\sum_I n_I \left(\Gamma(I \to {\rm aa}) + \frac{1}{2} \Gamma({\rm I} \to {\rm Xa}) \right)}{\sum_I n_I \Gamma(I \to {\rm X})} \,, \tag{27}$$

where n_I is the number density of the flaton I just before reheating.

If the flaton fields suffered negligible energy loss until they start their sinusoidal oscillation about their vev, one could in principle calculate the n_I by solving the field equation of motion under the potential Eq. (30). The same thing is possible if the energy loss can be calculated, the only known paradigm for that purpose being the parametric resonance approximation. Such a calculation would be difficult, and its uncertainty impossible to quantify at present.

One can however say something useful without knowing the n_I, by considering the quantities

$$B_I \equiv \frac{\Gamma(I \to {\rm aa}) + \frac{1}{2} ({\rm I} \to {\rm Xa})}{\Gamma(I \to {\rm X})} \,. \tag{28}$$

If all of these quantities were equal, they would be equal to $B_{\rm a}$. More usefully, if they are all known to have an upper or a lower bound r, in some regime of parameter space, then $B_{\rm a}$ has the same bound. In the case of an upper bound, one concludes from Eq. (26) that $\delta N_\nu < 4.4 r$, making the model in this regime *compatible* with a given bound on δN_ν if r is small enough. In the case of an upper bound, one concludes that $\delta N_\nu > 4.4 r$, making the model in this regime *incompatible* with a given bound if r is big enough.

3 Flaton and flatino spectrum

3.1 The superpotential

The model we consider contains two flaton superfields $\hat{P}$ and $\hat{Q}$, interacting with the superpotential [11, 12]

$$W_{\rm flaton} = \frac{f}{M_{\rm Pl}^n}\hat{P}^{n+2}\hat{Q}\,. \tag{29}$$

We deal with the simplest case $n = 1$ and assign the PQ charges -1 and 3 to $\hat{P}$ and $\hat{Q}$, respectively. Here we note that quantum gravity may break PQ symmetry, giving nonrenormalizable terms which invalidate the PQ solution to the strong CP problem. A way to avoid this is to impose a certain discrete gauge symmetry forbidding sufficiently higher dimensional operators [54]. However, to have such a discrete symmetry, one has to extend the model beyond the simple superpotential (29) under consideration. Our analysis can be applied to such extended cases with a straightforward generalization.

With the inclusion of the soft susy breaking terms and the cosmological constant, the potential is

$$V = V_0 + m_P^2|\phi_{\rm P}|^2 + m_Q^2|\phi_{\rm Q}|^2 + \frac{f^2}{M_{\rm Pl}^2}\left(9|\phi_{\rm P}|^4|\phi_{\rm Q}|^2 + |\phi_{\rm P}|^6\right) + \left(\frac{A_f}{M_{\rm Pl}}f\phi_{\rm P}^3\phi_{\rm Q} + h.c.\right) \tag{30}$$

The soft parameters m_P, m_Q and A_f are all of order 100 GeV in magnitude. It is assumed that m_P^2 and m_Q^2 are both positive at the Planck scale. The interactions of ϕ_P with the right handed neutrino superfields give radiative corrections which drive m_P^2 to a negative value at the PQ scale, generating vevs v_P and v_Q for respectively $|\phi_P|$ and $|\phi_Q|$. According to Eqs. (10) and (11), both vevs are roughly of order 10^{10} GeV. As we shall discuss in Section 3, the radial oscillations of the flaton fields ϕ_P and ϕ_Q correspond to two flatons, while the angular oscillations correspond to a third flaton and the axion.

3.2 Flaton spectrum

We write the flaton fields as

$$\begin{aligned}\phi_{\rm P} &= \frac{v_P + P}{\sqrt{2}}e^{i\frac{A_P}{v_P}} \\ \phi_{\rm Q} &= \frac{v_Q + Q}{\sqrt{2}}e^{i\frac{A_Q}{v_Q}}\,.\end{aligned} \tag{31}$$

From now on, we shall take v_P and v_Q as the independent parameters trading with m_P^2 and m_Q^2 in the potential Eq. (30). The vevs are taken to real and positive, and we shall take the other independent paramerters to be A_f and f to be real with opposite sign.

The axion field is

$$a = -\frac{v_P}{F_{PQ}}A_P + 3\frac{v_Q}{F_{PQ}}A_Q \tag{32}$$

where $F_{\rm PQ}^2 = v_P^2 + 9v_Q^2$. The orthogonal field to the axion (both are CP Odd) corresponds to a flaton. It is

$$\psi' = -\frac{v_P}{F_{\rm PQ}} A_Q - 3\frac{v_Q}{F_{\rm PQ}} A_P \,. \tag{33}$$

With our choice $A_f f < 0$, the vev is at $\psi' = 0$, and the mass-squared is

$$M_{\psi'}^2 = -\frac{f A_f v_P F_{\rm PQ}^2}{2M_{\rm Pl} v_Q} = -\frac{f}{g}\mu A_f \left(x^2 + 9\right) \tag{34}$$

where

$$\frac{\mu}{g} \equiv \frac{v_P v_Q}{2M_{\rm Pl}} \tag{35}$$

$$x \equiv \frac{v_P}{v_Q} \,. \tag{36}$$

For future convenience we have introduced a quantity μ, related to the g appearing only in the DFSZ model. At this stage results depend only on the ratio μ/g defined by (35) and they apply to both models.

The other two flatons correspond to the CP even fields P and Q. They have a $2\otimes 2$ mass matrix whose components are

$$\begin{aligned} M_{QQ}^2 &= M_{\psi'}^2 \frac{x^2}{9+x^2} \\ M_{PQ}^2 &= 9f^2 \frac{v_P^4}{M_{\rm Pl}^2 x} - 3\frac{M_{\psi'}^2 x}{9+x^2} = 3x\left(12\frac{f^2}{g^2}\mu^2 - \frac{M_{\psi'}^2}{9+x^2}\right) \\ M_{PP}^2 &= 3f^2 \frac{v_P^4\left(x^2+3\right)}{M_{\rm Pl}^2 x^2} - 3\frac{M_{\psi'}^2}{9+x^2} = 12\frac{f^2}{g^2}\left(x^2+3\right)\mu^2 - 3\frac{M_{\psi'}^2}{9+x^2} \,. \end{aligned} \tag{37}$$

Here two mass parameters m_P^2, m_Q^2 in Eq. (30) are replaced in favor of v_P, v_Q. Performing the rotation from the flavor basis $||P\ Q||$ to the mass basis $||F_1\ F_2||$

$$\begin{aligned} P &= \cos\alpha F_2 - \sin\alpha F_1 \\ Q &= \sin\alpha F_2 + \cos\alpha F_1 \,, \end{aligned} \tag{38}$$

we find the mixing angle α determined by

$$\begin{aligned} \cos 2\alpha &= \frac{M_{PP}^2 - M_{QQ}^2}{M_{F_2}^2 - M_{F_1}^2} = \epsilon\frac{x^2+3}{\sqrt{9+42x^2+x^4}} \\ \sin 2\alpha &= \frac{2M_{PQ}^2}{M_{F_2}^2 - M_{F_1}^2} = \epsilon\frac{6x}{\sqrt{9+42x^2+x^4}} \end{aligned} \tag{39}$$

where $\epsilon \equiv {\rm sign} M_{PQ}^2$ as we have the relation, $M_{F_2}^2 - M_{F_1}^2 = |M_{PQ}^2/3x|\sqrt{9+42x^2+x^4}$. Later, the decay rates can be expressed in terms of $\cos 2\alpha$ and $\sin 2\alpha$ without ambiguity in fixing the angle α itself. The two eigenstates F_1, F_2 have masses,

$$M_{F_{2,1}}^2 = \frac{\mu^2}{2}\left(\frac{f}{g}(12(x^2+3)\frac{f}{g} + (3-x^2)\frac{A_f}{\mu}) \pm |\frac{f}{g}(12\frac{f}{g} + \frac{A_f}{\mu})|\sqrt{9+42x^2+x^4}\right) \tag{40}$$

with $M_{F_2} > M_{F_1}$.

The requirement $M^2_{F_1} > 0$ gives the constraint

$$y_1 < y = -\frac{g\ A_f}{f\ \mu}\frac{9+x^2}{4x^2} < y_2\,, \tag{41}$$

where

$$y_{1,2} \equiv \frac{9+x^2}{8x^2}\left(21+x^2 \pm \sqrt{9+42x^2+x^4}\right)$$

or

$$\frac{1}{2}\left(21+x^2-\sqrt{9+42x^2+x^4}\right) < -\frac{g\ A_f}{f\ \mu} < \frac{1}{2}\left(21+x^2+\sqrt{9+42x^2+x^4}\right)\,. \tag{42}$$

One can find $M_{F_1} < M_{\psi'}$ for all the parameter space, and thus F_1 is the lightest flaton.

3.3 Flatino spectrum

From the superpotential W_{flaton} we can directly extract also the flatino's mass matrix whose eigenvalues are

$$M^2_{\tilde{F}_{2,1}} = \frac{9}{4}\frac{M^2_{\psi'}}{y\ x^2}[x^2+2\pm 2\sqrt{x^2+1}] = 9\frac{f^2}{g^2}\mu^2[x^2+2\pm 2\sqrt{x^2+1}] \tag{43}$$

The eigenstates $\tilde{F}_1, \tilde{F}_2$ are related to the flavor states $\tilde{P}, \tilde{Q}$ by

$$\begin{aligned} \tilde{F}_1 &= \cos\tilde{\alpha}\tilde{P} + \sin\tilde{\alpha}\tilde{Q} \\ \tilde{F}_2 &= -\sin\tilde{\alpha}\tilde{P} + \cos\tilde{\alpha}\tilde{Q} \end{aligned} \tag{44}$$

where the angle $\tilde{\alpha}$ is determined by

$$\begin{aligned} \cos 2\tilde{\alpha} &= -\frac{1}{\sqrt{1+x^2}} \\ \sin 2\tilde{\alpha} &= -\frac{x}{\sqrt{1+x^2}}\,. \end{aligned} \tag{45}$$

A parameter space analysis indicates that we have always $M_{F_1} \leq 2M_{\tilde{F}_1}$. This automatically forbids the decay of F_1 to flatinos leaving open only the decay into flatinos of the heavier F_2 and ψ' flatons.

4 Decays involving only flatons, flatinos and axions

In this section, we analyze the various decay rates between flatonic fields. We begin with the decay channels induced by the kinetic term and the superpotential W_{flaton}, which are common to the KSVZ and DFSZ models.

4.1 Derivative and cubic interaction terms between flatons

The flaton interaction terms with at least one derivative are given by the Lagrangian

$$\begin{aligned} L_\partial &= \frac{2\, v_P\, P + P^2}{2 v_P^2}[\frac{v_P^2}{F_{\rm PQ}^2}\,(\partial a)^2 + 9\frac{v_Q^2}{F_{\rm PQ}^2}\,(\partial \psi')^2 + 6\frac{v_Q v_P}{F_{\rm PQ}^2}\partial\psi'\partial a] + \\ & \frac{2\, v_Q\, Q + Q^2}{2 v_Q^2}[9\frac{v_Q^2}{F_{\rm PQ}^2}\,(\partial a)^2 + \frac{v_P^2}{F_{\rm PQ}^2}\,(\partial \psi')^2 - 6\frac{v_Q v_P}{F_{\rm PQ}^2}\partial\psi'\partial a] \end{aligned} \tag{46}$$

From this expression we can extract the following terms expressed in mass eigenstates:

(i) the trilinear derivative interactions with no axions,

$$(\partial\psi')^2\,\frac{1}{F_{\rm PQ} x\sqrt{x^2+9}}[\left(-9\sin\alpha + x^3\cos\alpha\right)F_1 + \left(9\cos\alpha + x^3\sin\alpha\right)F_2]\,; \tag{47}$$

(ii) the trilinear derivative interactions with only one axion,

$$L_{F_i\psi' a} = \partial\psi'\partial a \frac{6}{F_{\rm PQ}\,\sqrt{x^2+9}}[(\cos\alpha - x\sin\alpha)\,F_2 - (\sin\alpha + x\cos\alpha)\,F_1]\,; \tag{48}$$

(iii) the trilinear derivative interactions with two axions,

$$L_{F_i aa} = |\partial_\mu a|^2 \frac{1}{F_{\rm PQ}\sqrt{x^2+9}}\,((9\cos\alpha - x\,\sin\alpha)\,F_1 + (9\,\sin\alpha + x\,\cos\alpha)\,F_2)\,. \tag{49}$$

All the above derivative interactions can be transformed in scalar interactions if we are working at tree level and with on-shell external particles

$$\phi_1\,(\partial_\mu\phi_2)\,(\partial^\mu\phi_3) = \frac{1}{2}\left(M_{\phi_1}^2 - M_{\phi_2}^2 - M_{\phi_3}^2\right)\phi_1\phi_2\phi_3 \tag{50}$$

The cubic interactions come also from the superpotential and the soft terms

$$\begin{aligned} L_{\phi^3} &= \left(\frac{9}{2}f^2\frac{v_P v_Q^2}{M_{\rm Pl}^2} + \frac{5}{2}f^2\frac{v_P^3}{M_{\rm Pl}^2} - \frac{M_{\psi'}^2 v_Q x}{v_P^2\,(x^2+9)}\right)P^3 + \left(\frac{27}{2}f^2\frac{v_P^2 v_Q}{M_{\rm Pl}^2} - 3\frac{M_{\psi'}^2 x}{v_P\,(x^2+9)}\right)P^2 Q \\ &+ \left(\frac{9}{2}f^2\frac{v_P^3}{M_{\rm Pl}^2}\right)PQ^2 + \left(\frac{3f}{4}\frac{A_f F_{\rm PQ}^2}{M_{\rm Pl} v_Q}\right)P\psi'\psi' + \left(\frac{f}{4}\frac{A_f F_{\rm PQ}^2 v_P}{M_{\rm Pl} v_Q^2}\right)Q\psi'\psi'\,. \end{aligned} \tag{51}$$

In the mass basis, we get

$$\begin{aligned} L_{F_2F_1F_1} &= \Bigg\{-3\frac{M_{\psi'}^2\,\sin\alpha}{F_{\rm PQ} x\sqrt{9+x^2}}\,\left(-2x\cos^2\alpha + x\,\sin^2\alpha + \,\cos\alpha\,\sin\alpha\right) + \\ & \frac{6\,f^2\,\mu^2\sqrt{x^2+9}}{g^2 x F_{\rm PQ}}\Big(-18\,\cos^2\alpha\,\sin\alpha\,x + 9\,\sin^3\alpha\,x + 3\,\cos^3\alpha\,x^2 \\ & - \cos\alpha\,\sin^2\alpha(-9+x^2)\Big)\Bigg\}\,F_1^2 F_2 \equiv \frac{A_{F_2F_1F_1}}{2}F_1^2 F_2 \end{aligned} \tag{52}$$

For the full trilinear $F_2\psi'\psi'$ interaction, we have to add up the terms in Eqs. (47) and (51) to obtain

$$L_{F_2\psi'\psi'} = \frac{M_{F_2}^2}{2x\sqrt{9+x^2}F_{\rm PQ}}\left(-\left(3\sin\alpha + x\cos\alpha\right)\left(x^2+9\right)\frac{M_{\psi'}^2}{M_{F_2}^2}+\right. \tag{53}$$

$$\left.\left(9\cos\alpha+\sin\alpha x^3\right)\left(1-2\frac{M_{\psi'}^2}{M_{F_2}^2}\right)\right)F_2\psi'\psi' \equiv \frac{A_{F_2\psi'\psi'}}{2}\psi'^2 F_2$$

Collecting the above formulae one finds the decay rates among flatons and axions

$$\Gamma\left(F_2\to aa\right) = \frac{1}{32\pi}\frac{M_{F_2}^3}{F_{\rm PQ}^2\left(x^2+9\right)}\left(x\cos\alpha+9\sin\alpha\right)^2 \tag{54}$$

$$\Gamma\left(F_1\to aa\right) = \frac{1}{32\pi}\frac{M_{F_1}^3}{F_{\rm PQ}^2\left(x^2+9\right)}\left(-x\sin\alpha+9\cos\alpha\right)^2 \tag{55}$$

$$\Gamma\left(F_2\to F_1F_1\right) = \frac{1}{32\pi M_{F_2}}\sqrt{1-4\frac{M_{F_1}^2}{M_{F_2}^2}}|A_{F_2F_1F_1}|^2 \tag{56}$$

$$\Gamma\left(F_2\to \psi'\psi'\right) = \frac{1}{32\pi M_{F_2}}\sqrt{1-4\frac{M_{\psi'}^2}{M_{F_2}^2}}|A_{F_2\psi'\psi'}|^2 \tag{57}$$

$$\Gamma\left(F_2\to a\psi'\right) = \frac{1}{16\pi}\frac{M_{F_2}^3}{F_{\rm PQ}^2\left(x^2+9\right)}\left(1-\frac{M_{\psi'}^2}{M_{F_2}^2}\right)^3\left(3\cos\alpha-3x\sin\alpha\right)^2 \tag{58}$$

$$\Gamma\left(\psi'\to aF_2\right) = \frac{1}{16\pi}\frac{M_{\psi'}^3}{F_{\rm PQ}^2\left(x^2+9\right)}\left(1-\frac{M_{F_2}^2}{M_{\psi'}^2}\right)^3\left(3\cos\alpha-3x\sin\alpha\right)^2 \tag{59}$$

$$\Gamma\left(\psi'\to aF_1\right) = \frac{1}{16\pi}\frac{M_{\psi'}^3}{F_{\rm PQ}^2\left(x^2+9\right)}\left(1-\frac{M_{F_1}^2}{M_{\psi'}^2}\right)^3\left(3\sin\alpha+3x\cos\alpha\right)^2 \tag{60}$$

Energy conservation will of course forbid some of these reactions, depending on the flaton masses. As $M_{F_1} < M_{\psi'}$ the channels $F_1\to\psi'\psi'$ and $F_1\to\psi' a$ are always forbidden.

4.2 Interaction terms between flatons and flatinos

The trilinear Lagrangian terms responsible for the decay of flatons or flatinos are

$$\begin{aligned} L_{\phi\bar{\tilde\phi}\tilde\phi} &= \frac{3f}{2M_{\rm Pl}}\left(\left(v_PQ+v_QP\right)\bar{\tilde P}\tilde P + i\frac{v_P^2+3v_Q^2}{F_{\rm PQ}}\psi'\bar{\tilde P}\gamma_5\tilde P - i2\frac{v_Pv_Q}{F_{\rm PQ}}a\bar{\tilde P}\gamma_5\tilde P\right)+ \\ &\quad \frac{3f}{2M_{\rm Pl}}\left(2Pv_P\bar{\tilde P}\tilde Q + 2i\frac{v_P}{F_{\rm PQ}}\left(3v_Q\psi'+v_Pa\right)\bar{\tilde P}\gamma_5\tilde Q\right) \end{aligned} \tag{61}$$

(the tilded fields are the fermionic superpartner of the respective P and Q scalars). Let us denote the Yukawa couplings between the flaton (or the axion) and the flatinos in mass basis

by $-L_{Yuk} = Y_{ijk}\phi_i \bar{\tilde{F}}_j\,(1,\gamma_5)\,\tilde{F}_k/2$ where γ_5 is taken for $\phi_i = a, \psi'$. We find from Eq. (61) the following expressions for the Yukawa couplings

$$
\begin{aligned}
Y_{F_1\tilde{F}_1\tilde{F}_1} &= \frac{6f\mu\sqrt{x^2+9}}{gxF_{\rm PQ}}[(x\cos\alpha-\sin\alpha)\cos^2\tilde{\alpha}-x\sin\alpha\sin 2\tilde{\alpha}]\\
Y_{F_1\tilde{F}_2\tilde{F}_2} &= \frac{6f\mu\sqrt{x^2+9}}{gxF_{\rm PQ}}[(x\cos\alpha-\sin\alpha)\sin^2\tilde{\alpha}+x\sin\alpha\sin 2\tilde{\alpha}]\\
Y_{F_1\tilde{F}_1\tilde{F}_2} &= \frac{6f\mu\sqrt{x^2+9}}{gxF_{\rm PQ}}[-x\sin\alpha\cos 2\tilde{\alpha}+\frac{1}{2}\left(\sin\alpha-x\cos\alpha\right)\sin 2\tilde{\alpha}]\\
Y_{F_2\tilde{F}_1\tilde{F}_1} &= \frac{6f\mu\sqrt{x^2+9}}{gxF_{\rm PQ}}[(x\sin\alpha+\cos\alpha)\cos^2\tilde{\alpha}+x\cos\alpha\sin 2\tilde{\alpha}]\\
Y_{F_2\tilde{F}_2\tilde{F}_2} &= \frac{6f\mu\sqrt{x^2+9}}{gxF_{\rm PQ}}[(x\sin\alpha+\cos\alpha)\sin^2\tilde{\alpha}-x\cos\alpha\sin 2\tilde{\alpha}]\\
Y_{F_2\tilde{F}_1\tilde{F}_2} &= \frac{6f\mu\sqrt{x^2+9}}{gxF_{\rm PQ}}[+x\cos\alpha\cos 2\tilde{\alpha}-\frac{1}{2}\left(\cos\alpha+x\sin\alpha\right)\sin 2\tilde{\alpha}]\\
Y_{\psi'\tilde{F}_1\tilde{F}_1} &= \frac{6f\mu}{gxF_{\rm PQ}}[-\left(3+x^2\right)\cos^2\tilde{\alpha}-3x\sin 2\tilde{\alpha}]\\
Y_{\psi'\tilde{F}_2\tilde{F}_2} &= \frac{6f\mu}{gxF_{\rm PQ}}[-\left(3+x^2\right)\sin^2\tilde{\alpha}+3x\sin 2\tilde{\alpha}]\\
Y_{\psi'\tilde{F}_1\tilde{F}_2} &= \frac{6f\mu}{gxF_{\rm PQ}}[-3x\cos 2\tilde{\alpha}+\frac{1}{2}\left(3+x^2\right)\sin 2\tilde{\alpha}]\\
Y_{a\tilde{F}_1\tilde{F}_1} &= \frac{6f\mu}{gxF_{\rm PQ}}[2x\cos^2\tilde{\alpha}-x^2\sin 2\tilde{\alpha}]\\
Y_{a\tilde{F}_2\tilde{F}_2} &= \frac{6f\mu}{gxF_{\rm PQ}}[2x\sin^2\tilde{\alpha}+x^2\sin 2\tilde{\alpha}]\\
Y_{a\tilde{F}_1\tilde{F}_2} &= \frac{6f\mu}{gxF_{\rm PQ}}[-x^2\cos 2\tilde{\alpha}-x\sin 2\tilde{\alpha}] \qquad (62)
\end{aligned}
$$

From this we can extract the decay rates for $F_i \to \tilde{F}_j\tilde{F}_k$, $\psi' \to \tilde{F}_j\tilde{F}_k$, or $\tilde{F}_2 \to \tilde{F}_1 F_i\,(\psi', a)$

$$
\Gamma\left(F_i \to \tilde{F}_j\tilde{F}_k\right) = \frac{M_{F_i}}{8\pi}S\left(1-\frac{(M_{\tilde{F}_j}+M_{\tilde{F}_k})^2}{M_{F_i}^2}\right)^{\frac{3}{2}}\left(1-\frac{(M_{\tilde{F}_j}-M_{\tilde{F}_k})^2}{M_{F_i}^2}\right)^{\frac{1}{2}} Y^2_{F_i\tilde{F}_j\tilde{F}_k} \qquad (63)
$$

$$
\Gamma\left(\psi' \to \tilde{F}_j\tilde{F}_k\right) = \frac{M_{\psi'}}{8\pi}S\left(1-\frac{(M_{\tilde{F}_j}+M_{\tilde{F}_k})^2}{M_{\psi'}^2}\right)^{\frac{1}{2}}\left(1-\frac{(M_{\tilde{F}_j}-M_{\tilde{F}_k})^2}{M_{\psi'}^2}\right)^{\frac{3}{2}} Y^2_{\psi'\tilde{F}_j\tilde{F}_k} \qquad (64)
$$

$$
\begin{aligned}
\Gamma\left(\tilde{F}_2 \to \tilde{F}_1 F_i\right) &= \frac{M_{\tilde{F}_2}}{16\pi}\left(1-\frac{(M_{\tilde{F}_1}+M_{F_i})^2}{M_{\tilde{F}_2}^2}\right)^{\frac{1}{2}}\left(1-\frac{(M_{\tilde{F}_1}-M_{F_i})^2}{M_{\tilde{F}_2}^2}\right)^{\frac{1}{2}}\\
&\times \left(\left(1+\frac{M_{\tilde{F}_1}}{M_{\tilde{F}_2}}\right)^2-\frac{M_{F_i}^2}{M_{\tilde{F}_2}^2}\right)Y^2_{F_i\tilde{F}_1\tilde{F}_2} \qquad (65)
\end{aligned}
$$

$$\Gamma\left(\tilde{F}_2 \to \tilde{F}_1\psi'\right) = \frac{M_{\tilde{F}_2}}{16\pi}\left(1-\frac{(M_{\tilde{F}_1}+M_{\psi'})^2}{M_{\tilde{F}_2}^2}\right)^{\frac{1}{2}}\left(1-\frac{(M_{\tilde{F}_1}-M_{\psi'})^2}{M_{\tilde{F}_2}^2}\right)^{\frac{1}{2}}$$

$$\times \left(\left(1-\frac{M_{\tilde{F}_1}}{M_{\tilde{F}_2}}\right)^2-\frac{M_{\psi'}^2}{M_{\tilde{F}_2}^2}\right)Y^2_{\psi'\tilde{F}_1\tilde{F}_2} \tag{66}$$

$$\Gamma\left(\tilde{F}_2 \to \tilde{F}_1 a\right) = \frac{M_{\tilde{F}_2}}{16\pi}\left(1-\frac{M_{\tilde{F}_1}^2}{M_{\tilde{F}_2}^2}\right)\left(1-\frac{M_{\tilde{F}_1}}{M_{\tilde{F}_2}}\right)^2 Y^2_{a\tilde{F}_1\tilde{F}_2} \tag{67}$$

where S is a symmetric factor (1/2 for identical final states or otherwise 1).

5 Interaction of flatons and flatinos with matter fields

Now we study the interactions of the flatons with matter and supermatter. Through these interactions the flatons and flatinos decay into ordinary matter and axions, and the production of the latter must be sufficiently suppressed to satisfy the nucleosynthesis limit on the effective number δN_ν of extra neutrino species. (In this paper, we are for simplicity assuming that no flatino is stable.)

5.1 KSVZ model: Interactions between flatons and gluons

In the KSVZ (hadronic) model, the interaction with matter is

$$W_{\text{flaton-matter}} = h_{E_i}\hat{E}_i\hat{E}_i^c\hat{P} \tag{68}$$

where $\hat{E}_i$ and $\hat{E}_i^c$ are additional heavy quark and antiquark superfields.

The only decay mode available for the flatons is into two gluons coming from the anomaly (when the space phase will be available, we have to take into account also the decay into massive gluinos, in this discussion we neglect such a possibility). The respective one loop corrected decay rates are

$$\Gamma\left(F_1 \to g+g\right) = \frac{\alpha_S^2\left(M_{F_1}\right)}{72\pi^3}N_E^2\frac{M_{F_1}^3}{x^2\,F_{\rm PQ}^2}\left(x^2+9\right)\sin^2\alpha\left(1+\frac{95}{4}\frac{\alpha_S\left(M_{F_1}\right)}{\pi}\right)$$

$$\Gamma\left(F_2 \to g+g\right) = \frac{\alpha_S^2\left(M_{F_2}\right)}{72\pi^3}N_E^2\frac{M_{F_2}^3}{x^2\,F_{\rm PQ}^2}\left(x^2+9\right)\cos^2\alpha\left(1+\frac{95}{4}\frac{\alpha_S\left(M_{F_2}\right)}{\pi}\right) \tag{69}$$

where N_E is the total number of the superheavy exotic quark fields ($M_E = h_E v_P \gg M_{F_i}$). We do not consider the flatino decay into a gluon and a gluino which will be irrelevant for our discussion.

5.2 DFSZ model: Interactions between flatons/flatinos and ordinary matter

In the DFSZ model, the interaction is

$$W_{\text{flaton}-\text{matter}} = \frac{1}{2}\lambda \hat{N}\hat{N}\hat{P} + \frac{g}{M_{\text{Pl}}}\hat{H}_1\hat{H}_2\hat{P}\hat{Q} \tag{70}$$

where $\hat{N}$ are the right handed neutrino superfields and $\hat{H}_{1,2}$ the two Higgs doublets. Due to the second term we can provide a solution to the μ problem [55]. In such case we can add to the superpotential of the minimal supersymmetric standard model also the terms $h_\nu \hat{l}\hat{H}_2\hat{N}$ that generate the necessary mixing between left and right neutrinos to implement a see–saw mechanism which can explain the solar and atmospheric neutrino deficits.

The decay properties of the flatons now involve the direct interactions between flatons and ordinary matter and supermatter In general the interaction between flatons and Higgs fields are quite interesting due to the fact that these two sectors, after the spontaneous breaking of the PQ and the EW symmetry, mix together. We notice that the Peccei-Quinn symmetry prevents the introduction of a SUSY invariant mass term $\mu H_1 H_2$, solving automatically the so–called μ mass problem as mentioned before.

Let us start by writing the Higgs-flaton potential

$$\begin{aligned} V\left(H,\phi\right) \;=\;& |H_1|^2\left(m^2_{H_1} + \left|g\frac{\phi_{\text{P}}\phi_{\text{Q}}}{M_{\text{Pl}}}\right|^2\right) + |H_2|^2\left(m^2_{H_2} + \left|g\frac{\phi_{\text{P}}\phi_{\text{Q}}}{M_{\text{Pl}}}\right|^2\right) \\ &+\left\{gH_1H_2\left(A_g\frac{\phi_{\text{P}}\phi_{\text{Q}}}{M_{\text{Pl}}} + 3f^*\frac{\phi^{*2}_{\text{P}}|\phi_{\text{Q}}|^2}{M^2_{\text{Pl}}} + f^*\frac{\phi^{*2}_{\text{P}}|\phi_{\text{P}}|^2}{M^2_{\text{Pl}}}\right) + \text{c.c.}\right\} \\ &+\frac{1}{8}\left(g^2+g'^2\right)\left(|H_1|^2-|H_2|^2\right)^2 . \end{aligned} \tag{71}$$

When the fields $\phi_{\text{P,Q}}$ get vevs, the $m_3^2 H_1 H_2$ mass term is generated dynamically. The size of such a term is fixed by

$$m_3^2 = \mu\left(A_g + \frac{f}{g}\mu\left(x^2+3\right)\right) \tag{72}$$

In the limit $|m_3^2| \gg M_W^2$ the masses of the pseudoscalar A^0, of the CP even scalar Higgs field H^0 and of the charged Higgs fields $H^\pm$ are almost degenerate

$$M^2_{A^0,H^0,H^\pm} \simeq -\frac{m_3^2}{\sin\beta\cos\beta} \tag{73}$$

so from the constraint of positivity of such a masses we get

$$\frac{A_g}{\mu} + \frac{f}{g}(x^2+3) \leq 0 \tag{74}$$

In such a limit we also know that the mass eigenstate of the CP even electroweak sector H^0, h^0 and of the CP odd one A^0, G^0 are

$$\begin{aligned} H^0 &= -\sin\beta\, h_1^0 + \cos\beta\, h_2^0 \\ h^0 &= \cos\beta\, h_1^0 + \sin\beta\, h_2^0 \\ A^0 &= \sin\beta\, A_1^0 + \cos\beta\, A_2^0 \\ G^0 &= \cos\beta\, A_1^0 - \sin\beta\, A_2^0 \end{aligned} \tag{75}$$

where $H_1 = \frac{1}{\sqrt{2}}(v_1 + h_1^0 + iA_1^0)$ and $H_2 = \frac{1}{\sqrt{2}}(v_2 + h_2^0 + iA_2^0)$ are the gauge eigenstates and $\tan\beta = v_2/v_1$. To allow the flaton decay into A^0, we want it to be light so that small $\tan\beta$ is preferred in our discussion. Hereafter we will take $\tan\beta = 1$.

From Eq. (72), we find

$$\begin{aligned} V_{Fhh} = &\frac{1}{2}\mu^2\left(h_1^{02} + h_2^{02} + A_1^{02} + A_2^{02}\right)\left(\frac{P}{v_P} + \frac{Q}{v_Q}\right) + \frac{1}{2}\left[\left(h_1^0 h_2^0 - A_1^0 A_2^0\right) + i\left(h_1^0 A_2^0 + h_2^0 A_1^0\right)\right] \\ &\left[A_g\mu\left(\frac{P}{v_P} + \frac{Q}{v_Q} - i\frac{x^2+3}{xF_{\rm PQ}}\psi'\right) + 6\frac{f}{g}\mu^2\left(\frac{P}{v_P} + \frac{Q}{v_Q} + i\frac{3}{xF_{\rm PQ}}\psi'\right) + x^2\frac{f}{g}\mu^2\left(4\frac{P}{v_P} + i\frac{6}{xF_{\rm PQ}}\psi'\right)\right] \\ &+c.c. \end{aligned} \tag{76}$$

It is then simple manner to get the decay rates for the kinematically more favorable decay channels $F_{1,2} \to h^0 h^0$ and $\psi \to h^0 A^0$

$$\begin{aligned} \Gamma\left(F_1 \to h^0 h^0\right) &= \frac{M_{F_1}^3}{32\pi F_{\rm PQ}^2}\frac{(x^2+9)}{16\,x^2}\frac{\mu^4}{M_{F_1}^4}\left(1 - \frac{4M_{h^0}^2}{M_{F_1}^2}\right)^{1/2}|A_{F_1hh}|^2 \\ \Gamma\left(F_2 \to h^0 h^0\right) &= \frac{M_{F_2}^3}{32\pi F_{\rm PQ}^2}\frac{(x^2+9)}{16\,x^2}\frac{\mu^4}{M_{F_2}^4}\left(1 - \frac{4M_{h^0}^2}{M_{F_2}^2}\right)^{1/2}|A_{F_2hh}|^2 \\ \Gamma\left(\psi' \to h^0 A^0\right) &= \frac{M_{\psi'}^3}{16\pi F_{\rm PQ}^2}\frac{\mu^4}{M_{\psi'}^4}\left(1 - \frac{(M_{h^0} - M_{A^0})^2}{M_{\psi'}^2}\right)^{1/2}\left(1 - \frac{(M_{h^0} + M_{A^0})^2}{M_{\psi'}^2}\right)^{1/2}|A_{\psi hA}|^2 \end{aligned} \tag{77}$$

where

$$\begin{aligned} A_{F_1hh} &= \sin 2\beta[\left(\frac{A_g}{\mu} + 6\frac{f}{g}\right)(x\cos\alpha - \sin\alpha) - 4x^2\frac{f}{g}\sin\alpha] + 2\,(x\cos\alpha - \sin\alpha) \\ A_{F_2hh} &= \sin 2\beta[\left(\frac{A_g}{\mu} + 6\frac{f}{g}\right)(x\sin\alpha + \cos\alpha) + 4x^2\frac{f}{g}\cos\alpha] + 2\,(x\sin\alpha + \cos\alpha) \\ A_{\psi' hA} &= \left(\frac{A_g}{\mu} - 6\frac{f}{g}\right)\frac{(x^2+3)}{x} \end{aligned} \tag{78}$$

The flatino decay into ordinary particles comes from the superpotential $W = \frac{g}{M_{\rm Pl}}\hat{H}_1\hat{H}_2\hat{P}\hat{Q}$. We find that the flatino decay into a Higgs and a Higgsino (more precisely, the lightest neu-

tralino χ_1) has the rate;

$$\Gamma\left(\tilde{F}_i \to \chi_1 h^0\right) = \frac{M^3_{\tilde{F}_i}}{8\pi F^2_{\rm PQ}} \frac{\mu^2}{M^2_{\tilde{F}_i}} \left(x^2+9\right)^2 C^2_{\tilde{F}_i} \left(1 - \frac{\left(M_{\chi_1}+M_{h^0}\right)^2}{M^2_{\tilde{F}_i}}\right)^{1/2} \left(\left(1+\frac{M_{\chi_1}}{M_{\tilde{F}_i}}\right)^2 - \frac{M^2_{h^0}}{M^2_{\tilde{f}_I}}\right) \tag{79}$$

where $C_{\tilde{F}_1} = (\sin\tilde{\alpha} + x^{-1}\cos\tilde{\alpha})N_{\chi_1}$ and $C_{\tilde{F}_2} = (\cos\tilde{\alpha} - x^{-1}\sin\tilde{\alpha})N_{\chi_1}$. Here N_{χ_1} denotes the fraction of lightest neutralino in Higgsinos.

Let us now consider the flaton decay into ordinary fermions or sfermions. The mixing terms between flaton and Higgs fields allow a direct tree level coupling (after full mass matrix diagonalization) between the usual fermions and flatons. Parameterizing such a mixing with the parameter θ_{FH} the effective flaton-fermion interaction is $h_f\theta_{FH}$ so that the rate of decay is

$$\Gamma\left(F_i \to f+\bar{f}\right) = N_c \frac{h_f^2\theta^2_{FH}}{16\pi} M_{F_i}\left(1-4\frac{m_f^2}{M^2_{F_i}}\right)^{\frac{3}{2}} \tag{80}$$

where N_c is a color factor for the fermion f. Since $\theta_{FH} \simeq \left(\frac{v_{EW}}{F_{PQ}}\right)$

$$\Gamma\left(F_i \to f+\bar{f}\right)/\Gamma\left(F_i \to a+a\right) \sim h_f^2 v^2_{EW}/M^2_{F_i} \sim m_f^2/M^2_{F_i} \lesssim 1\,. \tag{81}$$

Therefore, the rate of the flaton decay into ordinary fermions cannot be made sufficiently larger than that into axions.

For the coupling between sfermions and flatons, we have two contributions. One is a direct coupling coming from the scalar potential

$$\begin{aligned} V_{F\tilde{f}\tilde{f}} = & \frac{\mu}{F_{\rm PQ}}\frac{\sqrt{x^2+9}}{x}\frac{v_1}{\sqrt{2}}\left(h_d\tan\beta\tilde{D}_L{}^*\tilde{D}_R{}^* + h_e\tan\beta\tilde{E}_L{}^*\tilde{E}_R{}^* + h_u\tilde{U}_L{}^*\tilde{U}_R{}^*\right) \\ & \left(F_1\left(x\cos\alpha-\sin\alpha\right)+F_2\left(\cos\alpha+x\sin\alpha\right)\right)+h.c. \end{aligned} \tag{82}$$

where $\tilde{D}^*$ denote down-type squark, *etc.*

The other arises from an indirect coupling induced by the mixing between Higgs and flaton fields as for the fermion case. Taking in consideration the cubic soft A-terms we find

$$V_{eff} = h_d A_d\,\theta_{F_i\,H_1}\,F_i\tilde{D}_L\tilde{D}_R + h_e A_e\,\theta_{F_i\,H_1}\,F_i\tilde{E}_L\tilde{E}_R + h_u A_u\,\theta_{F_i\,H_2}\,F_i\tilde{U}_L\tilde{U}_R \tag{83}$$

so that effectively we have couplings of the size

$$G_{F\,sfermion} \sim h_f(\mu+A_f)\frac{v_{\rm EW}}{F_{\rm PQ}} \tag{84}$$

Diagonalizing the sfermion mass matrix we can write $\tilde{f}_R\tilde{f}_L = a_{11}\tilde{f}_1\tilde{f}_1 + a_{22}\tilde{f}_2\tilde{f}_2 + a_{12}\tilde{f}_1\tilde{f}_2$ (where $a_{ii} \propto h_f$ so for $h_f \to 0$ we have $a_{12} \to 1$). Considering the decay of the light flaton we get

$$\Gamma\left(F_1 \to \tilde{f}_i + \tilde{f}_j\right) \simeq N_c \frac{G^2_{F\,\tilde{f}}}{64\pi\,M_{F_1}} a^2_{ij}\sqrt{1-4\frac{m^2_{\tilde{f}}}{M^2_{F_1}}} \tag{85}$$

Table 1: Direct decay channels involving only flatons ($I = F_1$, F_2 and ψ'), flatinos ($\tilde{F}_1$ and $\tilde{F}_2$), and the axion (a). The only decays which must occur are $F_i \to$ aa, $\psi' \to$ aF$_1$, and *either* $\psi' \to$ aF$_2$ *or* $F_2 \to \psi'$a. Any of the others may be forbidden by energy conservation, if the decaying particle is too light.

F_1	F_2	ψ'	$\tilde{F}_1$	$\tilde{F}_2$
aa	aa	aF$_1$	none	$\tilde{F}_1 I$
	aψ'	aF$_2$		
	$\tilde{F}_i\tilde{F}_j$	$\tilde{F}_i\tilde{F}_j$		
	F_1F_1			
	$\psi'\psi'$			

As observed in Ref. [15], the flaton may decay efficiently to two light stops as $h_t \sim 1$ and $a_{ij} \sim 1$ and thus a large splitting between light and heavy stops helps increasing the flaton decay rate to light stops. This kind of mass splitting occurs also in the Higgs sector and furthermore the light Higgs (h^o) is usually substantially lighter than the heavy Higgs (H^o) in the minimal supersymmetric standard model. This should be contrasted to the case with the mass splitting for stops which requires some adjustment in soft parameters. In this paper we concentrate on the flaton decay into Higgses, as the probably dominant mode, which in any case provides a lower bound on the decay rates to ordinary matter and therefore an upper bound on δN_ν.

6 Parameter space analysis

We have now evaluated all of the direct decay rates for flatons and flatinos into channels involving axions, as summarized in Table 1. We have also evaluated some of the contributions to the direct decay rates of flatons and flatinos into hadronic matter X. The ultimate objective is to evaluate the decay rates $\Gamma(I \to \mathrm{X})$, $\Gamma(I \to \mathrm{Xa})$, and $\Gamma(I \to \mathrm{aa})$ for each of the three flaton species, so as to evaluate δN_ν through Eqs. (26) and (27). For the reactions $F_i \to$ aa and $F_1 \to$ X, the direct rates are the same as the total rates, but in the other cases one has to consider also chain reactions. The reactions $F_2 \to$ X and $\psi' \to$ X can go either directly or through chains, and their rates are

$$\begin{aligned}
\Gamma(\psi' \to \mathrm{X}) &= \Gamma_{\mathrm{dir}}(\psi' \to \mathrm{X}) + \Gamma(\psi' \to \tilde{F}_1\tilde{F}_1) \\
&\quad + \Gamma(\psi' \to \tilde{F}_1\tilde{F}_2)B(\tilde{F}_2 \to \mathrm{X}) + \Gamma(\psi' \to \tilde{F}_2\tilde{F}_2)B^2(\tilde{F}_2 \to \mathrm{X}) \\
\Gamma(F_2 \to \mathrm{X}) &= \Gamma_{\mathrm{dir}}(F_2 \to \mathrm{X}) + \Gamma(F_2 \to \tilde{F}_1\tilde{F}_1) \\
&\quad + \Gamma(F_2 \to \tilde{F}_1\tilde{F}_2)B(\tilde{F}_2 \to \mathrm{X}) + \Gamma(F_2 \to \tilde{F}_2\tilde{F}_2)B^2(\tilde{F}_2 \to \mathrm{X})\,,
\end{aligned} \tag{86}$$

where B denotes a branching ratio. The reactions $\psi' \to \mathrm{aX}$, $F_2 \to \mathrm{aX}$ can go only through chains, and their branching ratios are

$$
\begin{aligned}
\Gamma(\psi' \to \mathrm{aX}) &= \Gamma(\psi' \to \mathrm{aF_2})\mathrm{B}(\mathrm{F_2} \to \mathrm{X}) + \Gamma(\psi' \to \tilde{\mathrm{F}}_1\tilde{\mathrm{F}}_2)\mathrm{B}(\tilde{\mathrm{F}}_2 \to \tilde{\mathrm{F}}_1\mathrm{a}) \\
&\quad + 2\Gamma(\psi' \to \tilde{F}_2\tilde{F}_2)B(\tilde{F}_2 \to \tilde{F}_1\mathrm{a})\mathrm{B}(\tilde{\mathrm{F}}_2 \to \mathrm{X}) \qquad (87)\\
\Gamma(F_2 \to \mathrm{aX}) &= \Gamma(F_2 \to \mathrm{a}\psi')\mathrm{B}(\psi' \to \mathrm{X}) + \Gamma(\mathrm{F_2} \to \tilde{\mathrm{F}}_1\tilde{\mathrm{F}}_2)\mathrm{B}(\tilde{\mathrm{F}}_2 \to \tilde{\mathrm{F}}_1\mathrm{a}) \\
&\quad + 2\Gamma(F_2 \to \tilde{F}_2\tilde{F}_2)B(\tilde{F}_2 \to \tilde{F}_1\mathrm{a})\mathrm{B}(\tilde{\mathrm{F}}_2 \to \mathrm{X}) \\
&\quad + 2\Gamma(F_2 \to \psi'\psi')B(\psi' \to \mathrm{aX})\mathrm{B}(\psi' \to \mathrm{X})\,.
\end{aligned}
$$

The rates for producing more axions are likely to be suppressed, because every term in the analogous expressions containing the product of more than two or more branching ratios.

Through Eqs. (26) and (27), these expressions provide the basis for a calculation of δN_ν, as a function of the parameters, if the relative values of the initial flaton densities n_I are known. Here, we perform the less ambitious task, of trying to identify a region of parameter space allowed by an upper bound δN_ν of order 1 to 0.1. (Recall that the former bound is roughly the present one [30], while improvements in the foreseeable future will either tighten the limit by an order of magnitude, or else detect a nonzero δN_ν.) We shall find that such a region probably does not exist in the KSVZ case, but that it certainly does exist in the DFSZ case.

6.1 Parameter space of KSVZ models

As discussed already, the flatons $F_{1,2}$ can decay into two gluons, so let us try to see if the generous requirements $B_{F_{1,2}} = \Gamma(F_{1,2} \to \mathrm{aa})/\Gamma(F_{1,2} \to gg) < 3$ [see Eq. (26)] can be fulfilled in any region of parameter space. From Eqs. (54), (55) and (69), we get

$$
\begin{aligned}
B_{F_1}^{-1} &\simeq 8 \times 10^{-4} N_E^2 \left(\frac{\sin\alpha\,(x^2+9)}{x\,(9\cos\alpha - x\sin\alpha)} \right)^2 \\
B_{F_2}^{-1} &\simeq 8 \times 10^{-4} N_E^2 \left(\frac{\cos\alpha\,(x^2+9)}{x\,(9\sin\alpha + x\cos\alpha)} \right)^2 \qquad (88)
\end{aligned}
$$

for $\alpha_S\,(M_F) \simeq 0.1$. With $\epsilon = +1$, we find that $B_{F_1}^{-1} < 7 \times 10^{-4} N_E^2$ for any value of x, and $B_{F_2}^{-1} < 6 \times 10^{-4} N_E^2/x^4$ in the limit $x \to 0$. Therefore, B_{F_2} can be made small enough for $x \sim 0.1$ but B_{F_1} cannot taking a reasonable value of N_E. A way out would come from a cosmological evolution of the flatons. That is, one could imagine a situation in which the flatons oscillate only along the direction of F_2 so that the populations of F_1 and ψ' (which can decay into $\mathrm{aF_1}$) are suppressed by the order of 10^4 compared to that of F_2. But this is not probable. On can find the similar behavior for $\epsilon = -1$ in which case B_{F_1} can be made small in the limit $x \to 0$.

6.2 Parameter space of DFSZ models

In this subsection, we will try to find a region of parameter space of the DFSZ models. For this, we consider rather stringent requirement; $B_I \lesssim 0.1$ for all three flatons and two flatinos.

According to Section 2.5, this requirement will ensure that $\delta N_\nu \lesssim 0.1$. Our strategy is to first write down a set of conditions which ensure the requirement, and then to identify a region of parameter space in which these conditions are satisfied.

6.2.1 The conditions

We first note that the decay rates calculated in the previous sections are functions of the 4 variables $x = v_P/v_Q$, f/g, A_f/μ and A_g/μ disregarding their overall dependence on $F_{\rm PQ}$. To be as independent as possible of the soft supersymmetry breaking parameters we will try to make analytic computations on the rates of the flaton decays into Higgs particles, in particular into the lightest Higgs boson (h^0) whose mass has an automatic upper bound of ~ 140 GeV [56]. We will concentrate on the region with $\frac{f}{g}$ negative and $\left|\frac{f}{g}\right| \ll 1$ and $x \gg 1$.

To open the decay channels of the flatons into Higgs particles we have, in particular, to impose $M_{\psi'} > M_A > 0$ which requires $\frac{A_g}{\mu} < \left|\frac{f}{g}\right| x^2$ with

$$\left|\frac{f}{g}\frac{A_f}{\mu}\right| x^2 > 2 \left|\frac{A_g}{\mu} + \frac{f}{g} x^2\right| . \tag{89}$$

The positivity of flaton masses requires

$$\left|\frac{A_f}{\mu}\right| < x^2 \left|\frac{f}{g}\right| . \tag{90}$$

In case the flatino production rates are sizable, we also impose $R_{\tilde{F}_2} \equiv \Gamma(\tilde{F}_2 \to \chi_1 h^0)/\Gamma(\tilde{F}_2 \to a\tilde{F}_1) > 10$ and $M_{\tilde{F}_1} > M_{\chi_1} + M_{h^0}$ to open the decay mode $\tilde{F}_1 \to \chi_1 h^0$. These two conditions give rise to the restrictions;

$$|f/g| < 0.02\, x\, N_{\chi_1} \quad \text{and} \quad |\frac{f}{g}| > \frac{1}{3\,x} . \tag{91}$$

For the latter condition, we required $M_{\tilde{F}_1} > \mu$.

Then we study, in our limit, the constraints given by the conditions $R_{\psi'} \equiv \Gamma(\psi' \to h^0 A^0)/\Gamma(\psi' \to aF_1) > 10$ and $R_{F_i} \equiv \Gamma(F_i \to h^0 h^0)/\Gamma(F_i \to aa) > 10$. The ratios R are

$$\begin{aligned} R_{\psi'} &\sim \frac{1}{144}\left(\frac{g}{f}\right)^2 \frac{\mu^2}{A_f^2}\left(\frac{A_g}{\mu} - 6\,\frac{f}{g}\right)^2 \\ R_{F_1} &\sim \frac{1}{4}\frac{\mu^4}{M_{F_1}^4}(\frac{A_g}{\mu} - 2\,\frac{f}{g}\,x^2 + 2)^2 \\ R_{F_2} &\sim 10^{-3}\,x^4 \frac{\mu^4}{M_{F_2}^4}\,(\frac{A_g}{\mu} + 18\,\frac{f}{g} + 2)^2 \end{aligned} \tag{92}$$

where $M_{F_1}^2 \sim |\frac{f}{g}\frac{A_f}{\mu}|x^2\mu^2$ and $M_{F_2}^2 \sim 12\,\frac{f^2}{g^2}x^2\mu^2$ for $\frac{A_f}{\mu} < 12|\frac{f}{g}|$, and $M_{F_1} \leftrightarrow M_{F_2}$ for $\frac{A_f}{\mu} > 12|\frac{f}{g}|$.

6.2.2 A viable region of parameter space

Now we identify a region of parameter space, in which the above conditions are all satisfied. Recall that we are considering only the region $x^2 \gg 1$, f/g negative, and $|f/g| \ll 1$. Within this region, we consider the four regions

$$
\begin{aligned}
I) &\quad \left|\frac{A_g}{\mu}\right| > \left|\frac{f}{g}\right| x^2 \\
II) &\quad 2 < \left|\frac{A_g}{\mu}\right| < \left|\frac{f}{g}\right| x^2 \\
III) &\quad \left|\frac{f}{g}\right| < \left|\frac{A_g}{\mu}\right| < 2 \\
IV) &\quad \left|\frac{A_g}{\mu}\right| < \left|\frac{f}{g}\right|
\end{aligned}
$$

Depending on $\frac{A_f}{\mu} < 12|\frac{f}{g}|$ or $\frac{A_f}{\mu} > 12|\frac{f}{g}|$ we define regions a or region b.

We find that all of the a regions are forbidden, and so is the IV_b region. The constraints for the other regions are as follows.

$$
\begin{aligned}
I_b &\quad x > 14, \quad A_g < 0, \quad 12\left|\frac{f}{g}\right| < \left|\frac{A_f}{\mu}\right| \\
&\quad 1 < \frac{1}{2}\left|\frac{A_f}{\mu}\right| < \left|\frac{f}{g}\right|\frac{x^2}{2} < \left|\frac{A_g}{\mu}\right| < 2\left(\left|\frac{f}{g}\right|\frac{x^2}{2}\right)^2 . \qquad (93) \\
II_b &\quad x > 9, \quad A_g > 0, \quad \left|\frac{f}{g}\right| < 3\times 10^{-2} \\
&\quad 2 < \left|\frac{A_f}{\mu}\right| < \left|\frac{f}{g}\right| x^2, \quad 2 < \left|\frac{A_g}{\mu}\right| < \left|\frac{f}{g}\right| x^2 . \qquad (94) \\
III_b &\quad A_g > 0, \quad 1 < 2\left|\frac{f}{g}\right| x^2, \\
&\quad 2\left|\frac{f}{g}\right| < \left|\frac{A_f}{\mu}\right|\left|\frac{f}{g}\right| < 2\times 10^{-2}, \qquad \left|\frac{f}{g}\right| < \left|\frac{A_g}{\mu}\right| < 2 . \qquad (95)
\end{aligned}
$$

In addition to these conditions, we need to add the original requirements, that f/g be negative with $|f/g| \ll 1$. (The latter condition is automatically satisfied in regions II_b and III_b, but not in region I_b.) The other original requirement $x \gg 1$ is automatically satisfied in all three regions. Note that all cases one requires $x^2|f/g| \gtrsim 1$.

Within each of these allowed regions, the parameters can take on their natural values $|A_f| \sim |A_g| \sim |\mu| \sim 100\,\text{GeV}$, $v_P \sim v_Q$ and $|f| \sim |g| \sim 1$, within a factor 10 or so.

7 Conclusions

We have explored the cosmology of a supersymmetric extension of the Standard Model, which has a Peccei-Quinn symmetry broken only by two 'flaton' fields ϕ_P and ϕ_Q. They have two radial modes of oscillation and one angular mode (plus the axion), corresponding to three flatons with mass of order 100 GeV, and there are two flatinos with roughly the same mass. The flatons are the generalizations of the saxion which appears in non-flaton models, and the flatinos are the generalizations of the axino.

The flaton models have the virtue that F_{PQ} is predicted in terms of the electroweak scale and the Planck or GUT scale. In the canonical model we have estimated $F_{\mathrm{PQ}} \sim 10^{10.4\pm0.9}\,\mathrm{GeV}$, and in the more complicated ones $F_{\mathrm{PQ}} \gtrsim 10^{11.8}\,\mathrm{GeV}$.

We have assumed that ϕ_P has a positive effective mass-squared in the early Universe, so that thermal inflation occurs, allowing rather definite predictions. The axion is a good dark matter candidate in all cases. (By *good* dark matter candidate we mean one whose density is predicted to be roughly in the right ballpark.) Bearing in mind that PQ strings are produced after thermal inflation, this simply corresponds to the received wisdom in the canonical case where $F_{\mathrm{PQ}} \sim 10^{10}\,\mathrm{GeV}$. But the axion is also a good candidate when F_{PQ} is bigger, because entropy production from flaton decay more than compensates for the increased axion density before flaton decay. Besides the axion, the LSP is also a good dark matter candidate in the canonical model. (In the other models, the reheat temperature is too low to thermalize the LSP, which must therefore be unstable.) In KSVZ models, a third good candidate is the supermassive particle whose mass comes mostly from a coupling to one of the flatons.

Our main concern has been with the highly relativistic axion population that is produced by flaton decay. It remains relativistic to the present and therefore makes no contribution to the dark matter, but it is dangerous for nucleosynthesis because it is equivalent to very roughly $\delta N_\nu \sim 1$ extra neutrino species. At present, the bound at something like the 2-σ level is $\delta N_\nu < 1.8$ for the 'high' deuterium nucleosynthesis scenario, and $\delta N_\nu < 0.3$ for the perhaps favored 'low' deuterium scenario. In the foreseeable future one will either have a bound $\delta N_\nu \lesssim 0.1$, or a detection of δN_ν.

We have calculated the rates for all relevant channels and examined the constraint that the energy density of these axions does not upset the predictions of the standard nucleosynthesis. We confirm the earlier conjecture, that the KSVZ case is probably ruled out even by the present bound $\delta N_\nu \lesssim 1$. For the DFSZ case there are more decay channels. To evade complicated phase space suppressions we concentrate on the decay of the flatons into Higgses, as the mass of the lightest Higgs boson has naturally a relatively low upper bound and the mass of the other Higgs boson and flaton fields are fixed by the parameters of the flatonic potential itself. In this way, we have found a region of parameter space, in which δN_ν will certainly be $\lesssim 0.1$, and in which the parameters can take on their natural values within a factor ten or so. In its DFSZ version, the flaton model of PQ symmetry breaking will be a candidate for explaining a future detection of nonzero δN_ν.

An interesting question, lying beyond the present investigation, is whether the allowed region of parameter space can be achieved in a supergravity model with universal soft parameters.

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